BRL

REPORT NO. 1492

STABILIZATION OF A LIQUID-FILLED SHELL BY INSERTING A
CYLINDRICAL PARTITION IN THE LIQUID CAVITY

by

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August 1970

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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IN THE LIQUID CAVITY

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B A L L I S T I C R E S E A R C H L A B O R A T O R I E S

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JTFrasier/WPD'Amico/lse
Aberdeen Proving Ground, Md.
August 1970

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IN THE LIQUID CAVITY

ABSTRACT

In July 1968, the XM613, a 107mm, WP mortar shell, was range tested
at Yuma Proving Ground. The projectile had a history of flight instabilities
when the WP was in the liquid state. Specially modified rounds incor-
porating a cylindrical partition mounted concentrically to the central
burster experienced stable flights. The rationale of inserting cylindrical
partitions in liquid-filled shell is explained as being a straightforward
and flexible method of changing internal geometries to produce stable flight
by means of understood design procedures.

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I. INTRODUCTION

Over the past decade investigators at the U.S. Army Ballistics Research Laboratories have conducted theoretical and experimental studies of the causes of flight instability of liquid-filled projectiles. Recently, these studies led to a rationale for the stabilization of such projectiles by incorporating within their payload compartment, an axially aligned cylindrical partition [1a]*. A limited, but successful, field test of the design rationale was made in July 1968.

The XM 613, a 107mm, spin stabilized, WP (white phosphorous) mortar shell, was range tested at Yuma Proving Ground. The projectile had a history of flight instabilities when the WP was in the liquid state. Specially modified rounds incorporating cylindrical partitions experienced stable flights. Limited resources allowed the testing of only two rounds, and therefore conclusive proof testing of the rationale cannot be claimed. However, the insertion of cylindrical partitions in liquid-filled projectiles is a straight-forward and flexible method to produce stable flight by means of well-founded design procedures. In fact it is the most flexible means for "a priori" design of a stable liquid-filled-projectile available to munitions designers today. This method is not seen as a panacea, but it does represent a major improvement over previous "ad hoc" procedures.

Our intention in the present paper is to explain the rationale for using a cylindrical partition to achieve well-behaved flights of liquid-filled shell. This is a simple task if we assume our audience is familiar with the work done at the BRL over the past ten years. This assumption is not likely to be valid, however. We will begin our presentation with a brief discussion of the problem of liquid-filled projectiles and our current understanding of the causes of their ills. Having done this, the use of cylindrical partitions in the payload compartment to achieve stability will be explained.

II. LIQUID-FILLED PROJECTILE STABILITY

WP rounds, one of the most common types of liquid-filled projectiles, have long been infamous for their poor flight behavior, (WP melts at 112°F). A major step toward understanding the physical realities of this behavior was achieved by Stewartson when he published

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* *Numbers in square brackets denote references found at the end of the paper.*

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an analysis of the stability of a spinning top containing liquid in a cylindrical cavity [2]. This work provided a clear definition of the basic mechanisms causing flight instabilities of liquid-filled projectiles and was the basis for further analytical and experimental research. A summary description of the current level of knowledge concerning liquid-filled projectiles is given very conveniently by using the conditions, assumptions, and results of Stewartson's analysis as a foundation. Theoretical and practical advances are, in large measure, the consequence of relaxation of the assumptions of Stewartson's analysis.

STEWARTSON'S THEORY OF STABILITY

Stewartson's theory concerns the flight stability of a spinning shell with a right circular cylindrical cavity either <u>wholly</u> or <u>partially</u> <u>filled</u> with liquid. Results of the theory show that <u>growth of the</u> <u>nutational</u> component of the projectile's yaw is possible under adverse combinations of the geometrical and physical characteristics of the projectile and its liquid filler. Instabilities are a consequence of the nutational frequency of the shell being hazardously close to certain natural frequencies of the liquid. This can be described as a condition of resonance. When this condition occurs, oscillations of the liquid produce a periodic moment (couple) on the shell casing and lead to a growth in yaw.

The theory is valuable for several reasons. First, it provides a clear understanding of the physical phenomena through which liquids produce instabilities in spinning projectiles, namely, the resonance type behavior mentioned above. This basic mechanism of instability applies to cavities of all geometries and not just cylinders. Second, the assumptions and conditions of the theory supply a useful framework to discuss the work at the BRL. The advances represented by the latter efforts are, in many instances, the consequence of modifications or relaxation of Stewartson's assumptions and conditions.

Stewartson's theory is based upon assumptions and stipulations that define the situations for which it is valid. Here we will review the most important of these factors and attempt to point out their physical significance. When appropriate, research advances by other investigators will be mentioned and references to their work cited. Certain of the stipulations are absolute requirements in that if they are not satisfied the theory is invalid. Others are taken as a matter of convenience to simplify the analysis and to clarify the role of the liquid in causing flight instability. An example of an assumption of the latter type is that the overturning moment is the only significant aerodynamic force or moment acting on the shell. Drag, etc., can be included in the analysis but are not essential to its development. Therefore, we shall neglect them to maintain focus on the basic features of interactions between the shell and its liquid. Distinctions between the two types of assumptions will become clear in the course of discussion.

Assumptions of Stewartson's Analysis

The Cavity in the Shell is a Right Circular Cylinder Whose Axis
is Parallel to the Spin Axis of the Projectile. Immediately
we see that the theory is restricted to shell with cylindrical cavities.
Hence, direct quantitative application of its results is limited to a
single geometrical shape. Wedemeyer, however, has achieved a modification
of Stewartson's theory through which it is possible to design for other
shapes; specifically, cavities whose radii change slowly along their
length [3a]. As a consequence, we are able to perform analyses on
many practical cavity goemetries. To use Wedemeyer's modification,
one must have a working knowledge of the basic Stewartson analysis.

The Aerodynamic Overturning Moment is the Only External Force
or Moment Affecting the Flight of the Shell. We explained
above that this assumption is taken for convenience. Gravity and
other aerodynamic effects can be taken into account through conventional
procedures but are not essential to the theory.

The Shell Flies with Constant Translational Velocity and Spin.
This assumption is consistent but implies that the liquid does
not influence the spin rate of the projectile. In practice these conditions
are never satisfied. For a period after the projectile leaves the gun,
the shell's spin decreases because it must spin-up the liquid. Subsequent
to liquid spin-up, the shell experiences spin decrease due to drag.
Furthermore, the projectile encounters translational drag. In general
however, once the transient phase of liquid spin-up complete, the drag
effects are sufficiently small for the current assumption to be reasonable
for most projectiles.

Finally, we remark that this assumption deals with the shell
casing - not with the liquid - and relates to the equation of motion
written for the projectile. Assumptions below concern the motion of
the liquid.

The Gross Motion of the Liquid is a Rigid Body Translation and
Spin Identical to the Translation and Spin of the Projectile.
This assumption, in conjunction with the one above, restricts our
considerations to the full spin condition of the liquid. The theory
does not consider situations where the shell casing and liquid have
unequal rigid body spins, nor does it take account of variations in
the spin of the liquid.

Wedemeyer [3b] and Scott [4] have performed analyses that allow
us to calculate the time required for the liquid to spin-up after a
projectile leaves the muzzle. Hence, in practical situations, we can
determine when Stewartson's full-spin assumption becomes valid. Usually,
it is soon after exit from the muzzle. Furthermore, a semi-empirical
analysis is available to determine whether instability is likely to
occur during the transient spin-up process [5].

The Spin of the Liquid and the Dimensions of the Cylindrical Cavity Satisfy the Condition.

$$a^2\Omega^2 >> qc \qquad (1)$$

where a = cavity radius, inches,

   Ω = spin rate of the projectile (and therefore the liquid), rad/sec,

   q = magnitude of the resolved gravity and drag vectors, in/sec^2,

   2c = height of the cavity, inches.

The physical significance of this assumption is that centrifugal forces exerted on the liquid due to its spin far outshadow any forces imposed by gravity or drag. A consequence of the assumption is that the liquid (except when the cavity is completely filled) has the shape of a cylinder with a hollow, cylindrical core*.

Equation (1) must be satisfied for Stewartson's theory to be used. If a shell experiences high drag along with a low spin rate there is a possibility the relation will be violated. Then the liquid will not have a cylindrical core, but will develop a paraboloidal surface. Designers should always verify Equation (1) is satisfied to avoid imprudent application of the theory.

It should be understood that the current assumption is not the external forces assumption. The former condition concerns external forces and moments acting on the shell casing and their effect on the motion of the shell. The present assumption concerns the effect of gravity and drag on the behavior of the liquid.

The Mass of the Liquid is Small Compared to the Total Mass of the Shell.
This assumption is one of convenience. It is satisfied for many shell and simplifies the equations of motion for the liquid-shell system. We use it here for these reasons.

The Liquid is Incompressible and Inviscid.
The assumption of incompressibility is reasonable for the liquids encountered in actual projectiles. Viscous effects, however, can influence the behavior of liquid-filled projectiles. Fortunately, Wedemeyer has provided an analysis to account for these effects [3c]. His analysis involves a boundary layer correction to the basic, inviscid theory of Stewartson.

The Final Assumptions Concern the Nature of any Variations to the Rigid Body Translation and Spin of the Shell and Liquid. Any Disturbance to the Shell's Motion is Restricted to Small Amplitude Perturbations Superposed on its Gross Translation and Spin. Correspondingly, the Liquid is Assumed to Experience only Small Amplitude Perturbations to its Large Scale Translation and Spin.
The assumption about the shell is the familiar small yaw situation associated with the linearized equations of yawing motion.

Actually a paraboloid whose vertex is far from the shell.

Similarly, the assumption imposed on the liquid linearizes the equations describing its behavior. By virtue of linearization, the equations for the liquid motions can be solved and their result incorporated into the equations of the motion for the shell.

Results of Stewartson's Analysis

All the basic assumptions and conditions underlying Stewartson's theory were presented above. From these assumptions, we can make a qualitative statement of our problem. Namely, determine the conditions for which a symmetric, rapidly spinning projectile will experience a flight instability as a consequence of having liquid (at full spin) in a cylindrical, axially symmetric cavity. Stewartson attacked this problem in two phases, and it seems most effective to describe his analysis in a similar fashion. First, he considered the behavior of the liquid in a state of rapid rotation within a container that could perform motions similar to those of the yawing motion of a shell. He then combined the problem solution he found for the liquid with the equations of motion for a shell. Upon analysis of the resulting equations it was found that under certain adverse conditions the yaw of the shell will grow without limit.

To describe the behavior of the liquid, we remember that it is confined in a container and that its basic motion involves rigid body spin about an axis with fixed direction. Upon assuming that the axis of the container is subjected to a small disturbance similar to the yawing motion of a shell, it is necessary that the liquid also experiences a disturbance to its basic motion because it must follow the walls of the cavity. Stewartson's solution shows that the liquid conforms to the cavity motion through the exitation of small amplitude oscillations superposed on the rigid body motion. There is an infinite number of discrete frequencies for these oscillations - the natural frequencies (or eigenfrequencies) of the spinning liquid. For an arbitrary motion of the container all the natural frequencies will be excited, but in varying degrees. If, however, the container performs a yawing motion at certain of the eigenfrequencies of the liquid, oscillations at this frequency become predominant, that is, a condition of resonance is established. As we shall describe later, it is this resonance that leads to the instability of a liquid filled projectile.

We should emphasize that the oscillations performed by the liquid are of small amplitude. Sloshing does not occur, but a wave pattern is established in the longitudinal, radial, and circumferential directions of the cavity and there are mode numbers* associated with each direction. For problems of projectile stability, an infinity of the possible longitudinal and radial modes are significant theoretically, but only the first circumferential mode is important. This circumstance is a result of the fact that the pressure fluctuations produced in the

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* *These can be thought of as fundamental wave patterns and harmonics.*

liquid by this mode lead to a periodic couple (with the same frequency as that of the oscillating liquid) on the walls of the container. It is this couple which renders a projectile unstable.

Now, we must explain how the natural frequencies of the liquid are determined. Stewartson's theory gives these frequencies in a complicated relation of the form

$$\tau_{nj} = \frac{\omega_{nj}}{\Omega} = f_n \left[\frac{c}{a(2j + 1)} , \frac{b^2}{a^2} \right]; \quad \begin{array}{l} n = 1, 2, 3, \ldots \\ \\ j = 0, 1, 2, \ldots \end{array} \quad (2)$$

where n = radial mode number (the number of nodes in the radial wave pattern),

j = longitudinal wave number ($2j + 1$ = number of nodes in the longitudinal wave pattern),

ω_{nj} = natural frequency of the nj^{th} node,

τ_{nj} = the non-dimensional eigenfrequency of the nj^{th} mode,

$2a$ = diameter of the cavity,

$2b$ = diameter of the cylindrical air core,

$2c$ = cavity length.

Several aspects of Equation (1) should be noted. First, the eigenfrequencies of the liquid are dependent upon the cavity geometry through the ratios c/a and b^2/a^2. The ratio b^2/a^2 is the air volume in the cavity expressed as a fraction of the total cavity volume. Hence $(1 - b^2/a^2)$ is the fraction of the cavity occupied by liquid. Next, we note that the eigenfrequencies depend upon the longitudinal mode number through the ratio $c/a(2j + 1)$ appearing as a variable in f_n. This is a fortunate circumstance, because once τ_{nj} is known for a set of fixed values of $c/a(2j + 1)$, b^2/a^2, and n, the eigenfrequencies are known for all longitudinal modes for which $c/a(2j + 1)$ equals the set value. Finally, Equation (2) shows the frequencies are linearly related to Ω, that is, τ_{nj} is independent of Ω.

As mentioned above, the function of f_n in Equation (1) is a complicated one, and it must be evaluated numerically through machine determination of the poles (singularities) of another equation appearing in the Stewartson analysis. This has been done and the results, the liquid eigenfrequencies, tabulated so that it is possible to make quantitive use of Stewartson's and subsequent work [1b].

This completes our discussion of the natural frequencies of the spinning liquid, which are dependant upon the cavity geometry, i.e., (c/a and b/a). Now, we turn to the questions of how and when

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an instability of a liquid-filled shell is produced by the oscillating liquid. To begin we recall our assumptions that the overturning moment is the only significant aerodynamic force or moment acting on the shell and that we are dealing with small yaws. Under these conditions, the motion of the shell (without the liquid) is governed by the relations

$$\lambda = \lambda_o e^{i\Omega\tau t} \tag{3}$$

$$I_y \tau^2 - I_x \tau + \frac{I_x^2}{4I_y s} = 0 \tag{4}$$

where I_x and I_y are, respectively, the axial and transverse moments of inertia of the shell, and

λ is the complex yaw,

t is time,

τ is the non-dimensional frequency of the motion of the shell,

s is the gyroscopic stability factor.

Equation (3) represents the form of the motion of the shell and the values of τ are provided by solution of Equation (4). Since Equation (4) is quadratic, the latter are found easily:

$$\tau_n = \frac{1}{2} \frac{I_x}{I_y} (1 + \sigma) \qquad \text{Nutational frequency} \tag{5}$$

$$\tau_p = \frac{1}{2} \frac{I_x}{I_y} (1 - \sigma) \qquad \text{Precessional frequency} \tag{6}$$

where $\qquad \sigma = \sqrt{1 - \frac{1}{s}}$,

$$s = \frac{\bar{\nu}^2}{4M} \quad,$$

$$\bar{\nu} = \frac{I_x}{I_y} \frac{2\pi}{n} \quad,$$

$$M = \frac{\rho_a Sd}{2m} K_t^{-2} C_{M_\alpha} \quad,$$

ρ_a = air density ,

S = reference area of shell, usually the cross-sectional area,

n = twist of rifling, calibers per turn,

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d = diameter of the shell ,

m = mass of the shell ,

$K_t^{-2} = md^2/I_y$,

C_{M_α} = aerodynamic overturning moment coefficient .

It is advantageous to recall here that the shell is stable (the yaw does not grow with time) so long as τ_n and τ_p are real quantities. To achieve this situation we must have s greater than one, a familiar condition for the gyroscopic stability of a projectile. If s is less than one, σ and therefore τ_n and τ_p become imaginary, and an exponential growth of yaw occurs.

Earlier, we stated that the oscillating liquid produces a moment on the casing of the projectile. Now we must describe that moment in functional form and modify Equation (4) to include its effect. Stewartson showed that when the frequency, τ, of the projectile is near any one of the natural frequencies, τ_{nj}, of the liquid, the moment applied to the shell casing is given by

$$\frac{M_L}{\Omega^2} = - \frac{\rho a^6 [2R(\tau_{nj})]^2/4c}{\tau - \tau_{nj}} \quad , \tag{7}$$

where M_L = the moment exerted on the shell by the liquid,

   τ = the frequency of motion of the shell,

   ρ = the density of the liquid,

   $R(\tau_{nj})$ = a (small) constant depending upon τ_{nj} and is always positive, "the residue".

The quantity R_{nj} is available in the tabulation of liquid eigenfrequencies referred to above [lb]. We see from Equation (7) that it governs the magnitude of the liquid-moment for a given cavity and frequency, τ_{nj}. Each possible frequency and modal configuration of the liquid involves a specific value of R_{nj}. With regard to the residue, it is convenient to point out a significant feature of its behavior. Namely, for any specific value of frequency, τ_{nj}, the residue decreases greatly for each successively larger value of n (i.e., R_{nj} decreases with increasing radial mode number). Thus, the higher radial modes produce relatively weak liquid-moments. In practice it has been found that modes beyond n = 2 are seldom strong enough to cause projectiles to be unstable.

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The moment due to the liquid is a forcing function on the motion of the shell. Thus to account for the presence of the liquid in the equation of motion of the shell we add Equation (7) to the right-hand-side of Equation (4) and obtain

$$I_y \tau^2 - I_x \tau + \frac{I_x^2}{4 I_y s} = - \frac{\rho a^6 [2R(\tau_o)]^2 / 4c}{\tau - \tau_o} \quad , \tag{8}$$

where for convenience, τ_o has been written in place of τ_{nj} to emphasize that we are now thinking of a specific fluid frequency. By solving Equation (8) for τ, the frequency of the shell's motion and the conditions under which the liquid can produce an unstable flight are determined, (that is, the conditions for which τ has an imaginary part assuming, of course, that $s > 1$). Here, we shall only summarize the results of this solution. It is found that when τ_o is close to τ_p, the precessional frequency, no instability occurs. However, if τ_o is near the nutational frequency, τ_n, Equation (8) has the roots

$$\tau = \left(\frac{\tau_n + \tau_o}{2}\right) \pm \sqrt{\left(\frac{\tau_n - \tau_o}{2}\right)^2 - \frac{\rho a^6 [2R(\tau_o)]^2}{4 c I_x \sigma}} \tag{9}$$

The condition for _instability_ is provided immediately by Equation (9). When the quantity under the radical is negative, τ has a negative imaginary part as we see by substituting Equation (9) into Equation (3):

$$\lambda = \lambda_o \exp\left\{ i\Omega\left(\frac{\tau_n + \tau_o}{2}\right)\tau - i\Omega \sqrt{\left(\frac{\tau_n - \tau_o}{2}\right)^2 - \frac{\rho a^6 (2R)^2}{4 c I_x \sigma}}\, t \right\} \tag{10}$$

$$= \lambda_o \exp\left[i\Omega\left(\frac{\tau_n + \tau_o}{2}\right)t \right] \exp(\alpha \Omega t) \quad ,$$

where $\quad \alpha = \sqrt{\frac{\rho a^6 (2R)^2}{4 c I_x \sigma} - \left(\frac{\tau_n - \tau_o}{2}\right)^2}$

Thus, an exponential growth of the nutational component of yaw occurs when

$$\left(\frac{\tau_n - \tau_o}{2}\right)^2 - \frac{\rho a^6 (2R)^2}{4 c I_x \sigma} < 0 \quad ,$$

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or, written more conveniently, when

$$- 1 < (\tau_o - \tau_n)/S^{1/2} < 1 \qquad \text{The condition for} \qquad (11)$$
$$\text{instability}$$

where S, "Stewartson's Parameter", is

$$S = \frac{\rho a^5 (2R)^2}{I_x \sigma (c/a)} \quad .$$

When Equation (11) is satisfied, the rate of growth of yaw is [Equation (10)].

$$\alpha = \frac{1}{2} \sqrt{S - (\tau_o - \tau_n)^2} \quad , \qquad (12)$$

or

$$\frac{2}{\sqrt{S}} \alpha = \sqrt{1 - \left(\frac{\tau_o - \tau_n}{\sqrt{S}}\right)^2}$$

Figure 1 is a plot of $(2/S^{1/2})\alpha$ against $(\tau_o - \tau_n)/S^{1/2}$. This curve, as well as examination of Equations (11) and (12), shows the yaw growth rate to be largest when $\tau_o = \tau_n$. For non-zero values of $\tau_o - \tau_n$, the yaw growth rate decreases until it vanishes for $|\tau_o - \tau_n| = S^{1/2}$.

Equations (11) and (12) are the basic results of the Stewartson theory. They permit us to calculate the conditions producing instability in a given projectile (and therefore a means to avoid these instabilities) and to calculate the strength of the instability, provided, of course, that the assumptions of Stewartson's analysis are satisfied. More accurately, we should say "provided that these conditions are satisfied or that advantage is made of the advances of Karpov [5], Wedemeyer [3] and Scott [4] to relax these restrictions." The work of these investigators was referenced during discussion of the assumptions of Stewartson's analysis. This work is extremely important in that it demonstrated the strengths and weaknesses of the original analysis and made the necessary advances for the practical application of the theory. Space does not permit any discussion of the details of this work. It can be summarized quickly, however, by pointing out that this work considers three points of concern that must be included in the analysis of any genuinely practical situation. Namely,

- Viscous effects in the liquid filler [3c],

- Certain types of non-cylindrical cavities, including those with profiles similar to the ogival shape of conventional artillery projectiles [3a],

- Liquid spin-up effects [3b].

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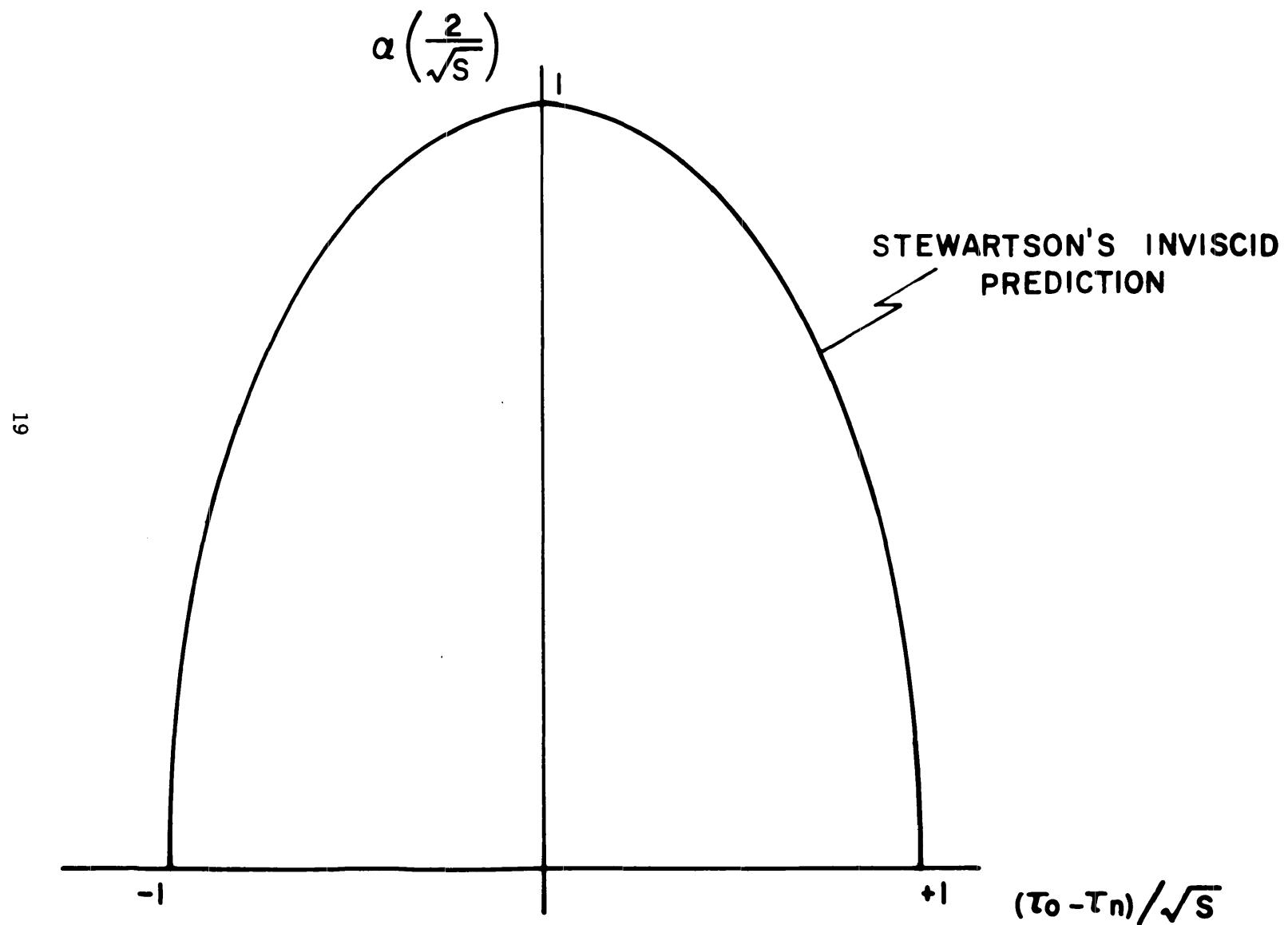
III. STABILITY OF A SHELL WITH A CENTRAL ROD ALONG ITS PAYLOAD COMPARTMENT

The preceeding discussion has centered about the problem of the stability of a projectile with a payload cavity that is either partially or completely filled with liquid. For the original Stewartson case this cavity is a right circular cylinder. However, owing to the efforts of Wedemeyer this restriction can be relaxed to treat cavities where a profile radius varies slowly with axial distance [2]. An important feature of these analyses, however, is that when the cavity is partially filled, a cylindrical air core runs along the axis of the cavity. A problem very similar to this and one that is important to the rationale for projectile design involves a core of rigid material along the center of the cavity rather than a flexible air core.

Let us substitute an axially rigid core for Stewartson's flexible air core. Analytical treatment of the problem of flight stability of a projectile carrying liquid in a payload compartment consisting of a right circular cylinder with an axially aligned rigid core is quite similar to that of the Stewartson problem. This problem has recently been solved by Frasier and Scott and has results completely analyogous to those discussed earlier [1c]. Namely, yaw of the projectile grows without limit if the nutational frequency of the projectile is sufficiently close to certain of the eigenfrequencies of the spinning liquid payload. More specificially, the constraints apply to the analysis; and the results expressed by Equation (1) through (12) apply with only one qualification. That qualification is that the dimension b used in those equations should be replaced by d, where d is the radius of the rod in the cavity. The residues for this case are numerically distinct from the air core solution.

We should emphasize that the analysis involving the rod requires that all the volume of the cavity after the rod is inserted is completely filled with liquid. Hence, the side walls, end faces, and rod are wetted by liquid at all times. We will know explain how all of the preceeding methods can be combined into procedures for the insertion of a cylindrical partition.

IV. RATIONALE FOR THE INSERTION OF A CYLINDRICAL PARTITION

The rationale for inserting cylindrical partitions in liquid-filled shell is an integrated use of Stewartson's theory and the subsequent extensions to techniques that account for non-cylindrical cavities, rigid central cores, and liquid spin-up. Once castings for a liquid-carrying projectile are made, it is not an easy task to modify the round for the elimination of shell-liquid resonances. Several "ad hoc" approaches for stabilization, longitudinal baffles being the most common, are frequently not successful. It is desirable to establish techniques that make possible "a priori" design control. A concept satisfying this criterion involves the insertion of a cylindrical partition in the payload cavity.

Flight instabilities due to liquid payloads are a consequence of resonance between liquid eigenfrequencies and the shell nutational frequency. This matching can occur during liquid spin-up or at a full spin condition. To alleviate such a resonant condition, the shell geometry, the percent of fill, or the shell nutational frequency must be changed. Normally, the liquid payload cannot be significantly altered. A redistribution of mass could shift the shell nutational frequency, but this results in a complete aerodynamic redesign. It appears that a variation in cavity geometry is the best route. This geometry modification should not be a haphazard one.

An intelligent change in geometry can be made by using a cylindrical partition. To illustrate this method, consider the two common shell cavities shown in Figure 2. Non-cylindrical cavities are shown with a burster located in the nose and with a central burster. Assume that the cavity is 95 percent filled with liquid. A cylindrical partition could be mounted about the longitudinal center line, as indicated by the broken lines. These cylinders must be fastened in such a way as to allow the liquid to move over the cylinder edges when it is thrown outward by centrifugal forces. Since the spinning liquid will seek the outermost position, the outer cavity will be 100 percent filled after the liquid spin-up process is completed.

Effectively two cavities result from the insertion of a cylinder in a shell cavity. For Figure 2a, the outer cavity takes up boundary conditions of a non-cylindrical wall and a rigid core (100 percent full). The inner cavity is the case treated by Stewartson, consisting of a partially filled cylindrical cavity [2]. A similar double cavity results in Figure 2b except that the central burster may act as a rigid core [1c] or as a partially wetted rod [6]. If the percent fill is not high enough to cause liquid-burster interference, then both Figure 2a and 2b have the same boundary conditions. Usually a cylinder radius can be selected that will result in a stable flight.

Assume that an analysis of some non-cylindrical cavity reveals a resonant condition. (How could a cylindrical partition be selected to stabilize the projectile?) In the case of the XM613, a 107mm, WP mortar shell, an unstable flight was caused by a transient resonance. The XM613 geometry is shown in Figure 3, with a cylindrical partition in place. It is possible to generate a table of nutational frequencies and rigid core radii that produce resonant conditions for the outer cavity [3a, 1c]. This is done for rigid core, full spin modes. Considering only the first radial mode, Table I lists the longitudinal modes (j = 2, 3, 4, and 5) for a fill ratio of 100 percent over a range of frequencies close to the XM613 nutational frequency of 0.067.
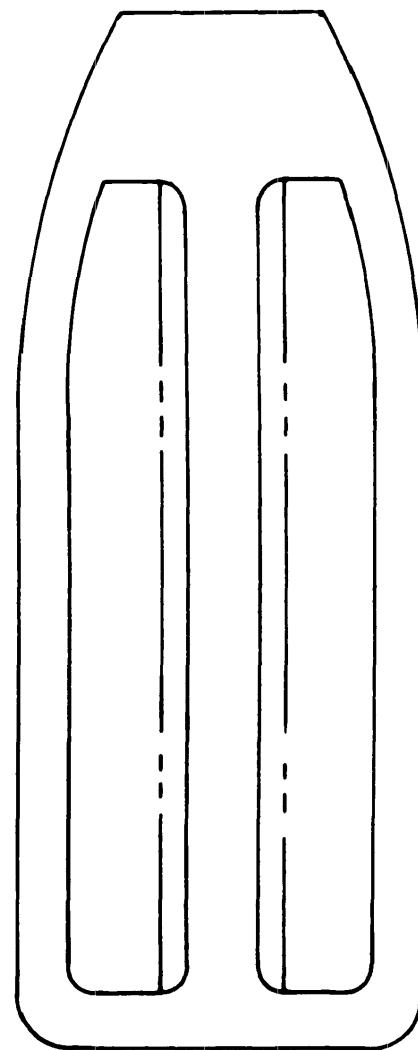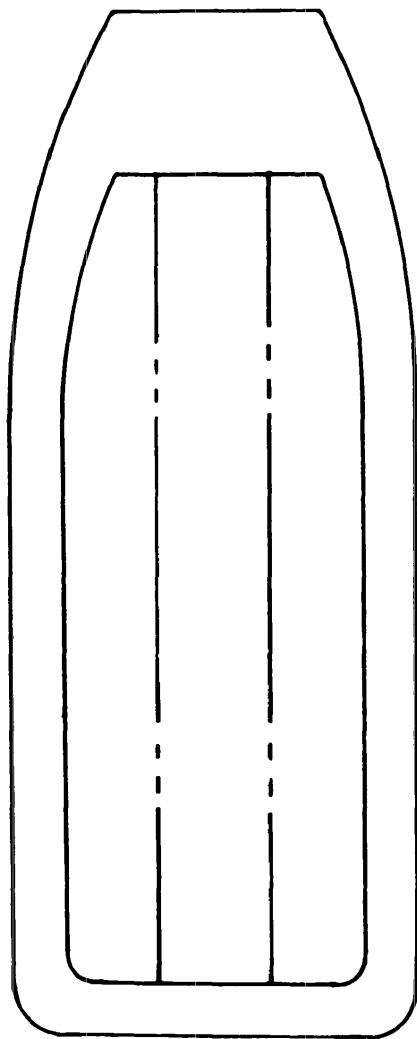
Figure 2. Typical Shell Cavities

.250

1.500

2 Slots
at 180°

Detail A

.250 D Holes
4 places

Section   B—B

9.75

B   B

.100

2.050 D

All dimensions in inches

0   1   2   3   4   5

PHYSICALS FOR XM613

Weight — 25.5 lbs.
Axial moment of inertia 64.0 lbs. in²
Transverse moment of inertia 740.0 lbs. in.²
Gyroscopic stability factor 1.43
Spin rate 68 radians / second
Fill ratio 95 %
Nutational frequency 0.067
Specific gravity 1.72
Kinematic viscosity 1.2 centistokes

Figure 3. Physical Characteristics of the XM613

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τ	$d_{j=2}$ inches	$d_{j=3}$ inches	$d_{j=4}$ inches	$d_{j=5}$ inches
0.00	0.568	0.891	1.036	1.138
0.02	0.616	0.910	1.052	1.156
0.04	0.653	0.927	1.066	1.166
0.06	0.686	0.944	1.081	1.179
0.08	0.718	0.961	1.096	1.194
0.10	0.745	0.978	1.111	1.208

A graphical representation of Table I is shown in Figure 4. The selection of a nonresonant cylinder radius, d, can easily be made. Logical choices would be 0.85" or 1.025". If values of 0.70" or 0.95" were selected, a resonant condition is produced. Once d and a cylinder wall thickness are chosen then the geometry of the inner cavity is fixed. For the XM613, a cylinder outside radius of 1.025" and a wall thickness of 0.10" were selected. The slenderness ratio (height/diameter) of the inner cavity for the j = 0 mode was 5.27. The resulting percent of fill for the inner cavity was 62.5 percent.

The new design was checked for viscous effects and transient resonances [3,7]. Conditions for the XM613 were such that no viscous corrections needed to be made. Transient resonances can occur in either of the cavities. Transient eigenfrequencies cannot be explicitly calculated, but the yaw growth while passing through a transient instability can be approximated by the following equation [5,7].

$$\log\left(\frac{\alpha_1}{\alpha_0}\right) = -\left(\frac{\pi\Omega}{4}\right)\frac{S}{d\tau_0/dt} \quad .$$

where α_0 = yaw angle before transient resonance,

α_1 = yaw angle after transient resonance,

Ω = shell spin rate,

S = Stewartson's parameter,

τ_0 = time dependent, non-dimensional eigenfrequency,

t = time.
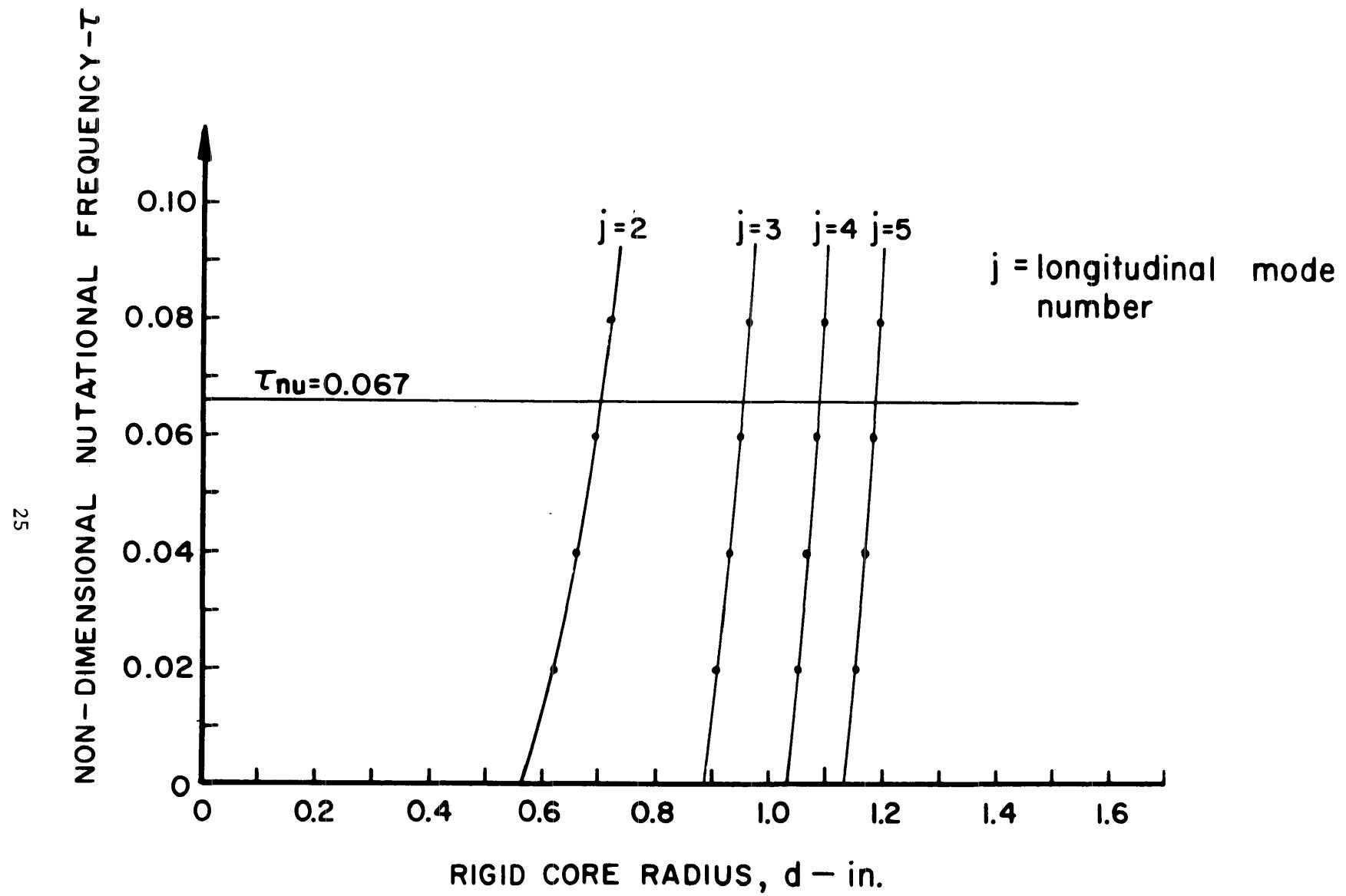
Figure 4. Nondimensional Frequencies Versus Partition
Outside Radii

If $d\tau_0/dt$ is computed correctly, viscous effects in the spin-up process are taken into account. Assumptions made to extend the above equation to partially filled cylinders or to non-cylindrical, solid core cavities are not overly restrictive. For the inner cavity of the XM613, a transient resonance was located for an effective percent fill of 0.597. Since α_1/α_0 was calculated to be 1.35, no serious problems were expected.

It is quite possible the cylinder radii available from plots such as Figure 4 could still yield serious resonances for either spin-up or full spin conditions. It might then be possible to use more than one cylinder, especially in some of the larger rounds. A multi-cylinder design should still allow for fluid movement, while not decreasing the payload capacity or impairing the manufacturing process. This was not needed for the XM613, which was flown just as shown in Figure 3.

Firings held in July 1968 at Yuma Proving Ground were part of a project termination that included the expenditure of available hardware. Since the XM613 had a history of flight instabilities, blamed upon shell-liquid resonances, this afforded the design engineers a test vehicle for "ad hoc" stabilization methods. At the invitation of personnel from the Artillery and Mortar Section of Picatinny Arsenal and the Weapons Development and Engineering Laboratory of Edgewood Arsenal, the BRL suggested the use of a cylindrical partition. Other modifications made by the MUCOM agencies included the insertion of several types of longitudinal baffles, the use of a gel filler representing thickened WP, and the addition of a sponge material into the shell cavity in an attempt to suspend the WP, i.e., reduce fluid movement. Firings were made from a single tube for a single charge and elevation. Thirty projectiles were thermally cured to a surface temperature of 145°F for a period of twenty-four hours. Four to five rounds were removed from the conditioning oven and fired within thirty minutes until all ammunition was expended. This insured that the filler was in a liquid state. Of the thirty rounds fired only the two rounds that employed cylindrical partitions flew well. These two rounds fell ten meters apart at 6,000 meters.

V. CONCLUSIONS

The small number of rounds that have actually been field tested does not represent statistical proof or conclusive verification of the overall ability of a cylindrical partition to stabilize liquid-filled shell. It does, however, produce a high level of confidence in the technique and in the combined use of many theoretical methods. The use of a cylindrical partition is the only available procedure that has a well-understood theoretical foundation for "ad hoc" stabilization.

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13. ABSTRACT

In July 1968, the SM613, a 107mm, WP mortar shell, was range tested at Yuma Proving Ground. The projectile had a history of flight instabilities when the WP was in the liquid state. Specially modified rounds incorporating a cylindrical partition mounted concentrically to the central burster experienced stable flights. The rationale of inserting cylindrical partitions in liquid filled shell is explained as being a straightforward and flexible method of changing internal geometries to produce stable flight by means of understood design procedures.

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